Model Paper

GOVERNMENT COLLEGE UNIVERSITY, FAISALABAD

BA/B.Sc (Part-I) Annual 2017

Subject: Mathematics (B-Course) Paper: (I) Course Code: MTH-302

Course Title: Vector Analysis and Mechanics

Time Allowed: 03:00 Hours Maximum Marks: 100 Pass Marks: 33%

Note: Attempt Six questions in all, selecting two questions from section II & III each and One question from I & IV.

SECTION-I

Q. No. 1 (a) Establish the identity $[a\times l\ b\times m\ c\times n]+[a\times m\ b\times n\ c\times l]+[a\times n\ b\times l\ c\times m]=0$ **(9,8)**

(b) If **a** is a constant vector and $\mathbf{r}=x\mathbf{i}+y\mathbf{j}+z\mathbf{k}$ show that $Curl[(\mathbf{a.r})\mathbf{r}]=\mathbf{0}$

Q. No. 2 (a) Find curl(grad ϕ), where ϕ is a scalar function. **(9,8)**

(b) Prove that $\nabla\times(\nabla\times\boldsymbol{F})=\nabla(\nabla.\boldsymbol{F})-\nabla^2\boldsymbol{F}$.

SECTION-II

Q. No. 3 (a) A system of forces act on a plate in the form of an equilateral triangle of side 2a . The moments **(9,8)** of the forces about three vertices are G_1, G_2 and G_3 respectively. Show that the magnitude of their resultant is $\sqrt{\dfrac{G_1{}^2+G_2{}^2+G_3{}^2-G_1G_2-G_2G_3-G_1G_3}{3a^2}}$.

(b) If forces $l\mathbf{AB}, m\mathbf{BC}, l\mathbf{CD}, m\mathbf{DA}$ acting along sides of a quadrilateral triangle are equivalent to a couple, show that either $l=m$ or $ABCD$ is a parallelogram.

Q. No. 4 (a) A triangular lamina ABC, right angled at A, rests with its plane vertical, and with the sides AB, AC supported by smooth fixed pegs D, E in a horizontal line. Prove that the inclination θ of AC to the horizontal is given by $AC\cos\theta-AB\sin\theta=3DE\cos2\theta$. **(9,8)**

(b) AB and AC are similar uniform rods, of length a, smoothly joined at A. BD is a weightless bar of length b, smoothly joined at B, and fastened at D to a smooth ring sliding on AC. The system is hung on a small smooth pin at A. Show that the rod AC makes the vertical an angle $\tan^{-1}\left(\frac{b}{a+\sqrt{a^2-b^2}}\right)$.

Q. No.5 (a) Show that the centre of mass of a segment of a solid sphere of radius a at a distance b from the center of the sphere is at a distance $\dfrac{3(a+b)^2}{4(2a+b)}$ from the center. **(9,8)**

(b) The radius of the faces of a frustum of a solid cone are 2 ft. and 3 ft. and the height of the frustum is 4 ft. Find the distance of center of gravity from the larger face.

Q. No.6 (a) A rod 4 feet long, rests on a rough floor against the smooth edge of a table of 3 feet hight. If the rod is on the point of slipping when inclined at an angle of 60° to the horizontal, find the coefficient of friction. **(9,8)**

(b) A Solid cylinder rests on a rough horizontal plane with one of its flat ends on the plane, and is acted on by a horizontal force through the center of its upper end. If this force be just sufficient to move the solid show that it will slide, and not topple over, if the coefficient of friction be less than the ratio of radius of the base of the cylinder to its height.

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SECTION-III

Q. No. 7 (a) A particle describing simple harmonic motion has velocities 5 ft/sec. and 4 ft/sec. **(8,8)** when its distance from the centre are 12 ft. and 13 ft. respectively. Find the time period of the motion.

(b) A particle is projected vertically upward with velocity $\sqrt{2gh}$ and another is let fall from a height h at the same time. Find the height of point where they meet each other.

Q. No. 8 (a) An aero plane is flying with constant speed $v_\circ$ and at constant height h. Show that, **(8,8)** if the gun is fired point blank at the aero plane after it has passed directly over the gun when its angle of elevation as seen from the gun is α the shell will hit the aero plane provided $2(V\cos\alpha - v_\circ)v_\circ\tan^2\alpha = gh$, where V is the initial speed of the shot, the path being assumed parabolic.

(b) A projectile having horizontal range R, reaches a maximum height H. Prove that it must have been launched with (i) an initial speed is equal to $\sqrt{\frac{g(R^2+16H^2)}{8H}}$, and (ii) at an angle with the horizontal given by $\sin^{-1}\left(\frac{4H}{\sqrt{(R^2+16H^2)}}\right)$.

Q. No. 9 (a) If a Particle P moves with a velocity v given by $v^2 = n^2(ax^2+2bx+c)$, show that particle **(8,8)** P executes a simple harmonic motion. Find the centre, the amplitude and the time period of the motion.

(b) A particle describing simple harmonic motion in such a way that its velocity and acceleration at a point P are u and f respectively and the corresponding quantities at another point Q are v and g Find the distance PQ.

Q. No. 10 (a) A particle of mass m moves under a central force $mM3au^4 - 2(a^2-b^2)u^5\ (a>b)$ **(8,8)** and is projected from an apse at distance $a+b$ with velocity $\frac{\sqrt{M}}{a+b}$. Show that orbit is $r = a + b\cos\theta$.

(b) A palnet is describing an ellipse about the sun as focus. Show that its velocity away from the sun is greatest when the radius vector is to the planet is at right angles to the major axis of the path, and it then is $\frac{2\pi ae}{\tau(1-e^2)}$, where $2a$ is the major axis, e the eccentricity and τ is the periodic time.

SECTION-IV

Q. No. 11 (a) From a point on a smooth horizontal plane, a ball is projected with velocity u at an **(9,8)** angle α to the horizon. Show that it will keep rebounding from the plane for a time $\frac{2u\sin\alpha}{g(1-e)}$ and will have a range $\frac{u^2\sin\alpha}{g(1-e)}$, where e is the coefficient of restitution.

(b) A smooth sphere of mass m impinges on another of mass M at rest, the direction of motion making an angle 45° with the line of centres at the moment of impact. Show that if $e = 1/2$, the direction of motion of the sphere of mass m is turned through an angle $\tan^{-1}\left(\frac{3m}{4m+M}\right)$.

Q. No. 12 (a) A ball impinges directly upon another ball at rest and is itself reduced to rest by the **(9,8)** impact. If half of the initial kinetic energy is destroyed in the collision, find the coefficient of restitution.

(b) A ball A of mass m, moving with velocity u, impinges directly on an ball B of same mass m moving with velocity v in the opposite direction. If A be brought to rest by the impact show that $u : v = 1+e : 1-e$. where e is the coefficient of restitution.

Model Paper

GOVERNMENT COLLEGE UNIVERSITY, FAISALABAD

B.A/B.Sc (Part-II) Annual 2017

Subject: Mathematics (B-Course) Paper: <u>(II)</u> Course Code: MTH-402

Course Code: Mathematical Methods, Group Theory & Metric spaces

Time Allowed: 03:00 Hours Maximum Marks: 100 Pass Marks: 33%

Note: Attempt Six questions in all, selecting two questions from section I & II each and One question from III & IV.

SECTION-I

Q.1 (a) Prove that $\left(\frac{1+\sin x+i\cos x}{1+\sin x-i\cos x}\right)^n=\cos n\left(\frac{\pi}{2}-x\right)+i\sin n\left(\frac{\pi}{2}-x\right)$. **(9,8)**

(b) Sum the series to infinity. $\cos\theta-\frac{1}{2}\cos 2\theta+\frac{1}{3}\cos 3\theta-\frac{1}{4}\cos 4\theta+\ldots$

Q.2 (a) Prove that $\tan^{-1}\left(\frac{x+iy}{x-iy}\right)=\frac{\pi}{4}+\frac{i}{2}\ln\frac{x+y}{x-y}$, if $x>y>0$. **(9,8)**

(b) Find the direction of Qibla of SHAH FAISAL MASJID, Islamabad, given that

Latitude of Islamabad = $33^\circ-40'\ N$, Longitude of Islamabad = $73^\circ-8'\ E$.

Q.3 (a) Examine the continuity of $f(x,y)=\begin{cases}\frac{x^2y}{x^4+y^2}, & if\ (x,y)\neq(0,0)\\ 0, & if\ (x,y)=(0,0)\end{cases}$ at $(0,0)$.

Also find $f_x(0,0)$ and $f_y(0,0)$, if exists. **(9,8)**

(b) If $f(x,y)=x^2\tan^{-1}\left(\frac{y}{x}\right)-y^2\tan^{-1}\left(\frac{x}{y}\right)$, then show that $\frac{\partial^2 f}{\partial x\partial y}=\frac{x^2-y^2}{x^2+y^2}$.

Q.4 (a) If $u=f(x,y)$ is homogeneous function of degree n, then prove that

$x^2f_{xx}+2xyf_{xy}+y^2f_{yy}=n(n-1)f$. **(9,8)**

(b) Examine $f(x,y)=2x^2-4x+xy^2-1$ for relative extrema.

SECTION-II

Q.5 (a) Prove that the series $\sum_{n=1}^{\infty}\frac{1}{n^p}$ is divergent when $p\leq 1$ and convergent when $p>1$. **(9,8)**

(b) Use alternating series test to check the series $\sum_{n=1}^{\infty}(-1)^{n-1}\left(\frac{n+4}{n^2+n}\right)$ for convergence or divergence.

Q.6 (a) State Cauchy's root test for convergence or divergence of infinite series apply, this test

on $\sum_{n=1}^{\infty}\frac{2^n}{n^3}$ to check convergence or divergence. **(9,8)**

(b) Find the radius of convergence and the interval of convergence of the power series

$\sum_{n=0}^{\infty}\frac{(-1)^{n+1}(x+1)^{2n}}{(n+1)^2 5^n}$.

P.T.O

Q.7	(a)	Evaluate $\iint_D \frac{x^2}{(x^2+y^2)^2} dA$, where D is the region in the first quadrant enclosed by the circles $x^2+y^2=a^2$, $x^2+y^2=b^2$; $< a < b$.	(9,8)
	(b)	Find the volume of a right pyramid whose height is ***h*** and has a square base with each side of length ***a***.	
Q.8	(a)	Evaluate the improper integral $\int_0^1 x\ln x \, dx$.	(9,8)
	(b)	Evaluate $\iiint_S 15x^2z^2 dxdydz$, where S is bounded by $x^2+y^2=1$ and $x^2+z^2=1$.	

SECTION-III

Q.9	(a)	Let $(G,\cdot)$ be a group. Such that $(ab)^n = a^n b^n$ for three consecutive natural numbers and all $a,b \in G$. Show that G is an abelian group.	(8,8)
	(b)	The union $H \cup K$ of two subgroups H and K of a group G is a subgroup of G if and only if either $H \subset K$ or $K \subset H$.	
Q.10	(a)	If H and K be two finite subgroups of a group G, whose order are relatively prime, prove that $H \cap K = \{e\}$.	(8,8)
	(b)	Define a transposition and determine whether the permutation $\begin{pmatrix} 1\,2\,3\,4\,5\,6\,7 \\ 4\,3\,1\,2\,6\,7\,5 \end{pmatrix}$ is even or odd.	

SECTION-IV

Q.11	(a)	Let (X,d) be a metric space and let $d'(X \times X) \to R$ be given by $d'(x_1,x_2) = \frac{d(x_1,x_2)}{1+d(x_1,x_2)}$. Prove that d' is a metric space.	(8,8)
	(b)	Prove that open sphere in a metric space is an open set.	
Q.12	(a)	Define the following in the metric space X i) Closed sphere ii) Open set iii) Closure of a set A in X iv) Limit point of a set A in X	(8,8)
	(b)	If A and B are two subsets of a metric space (X,d), then prove that i) $\text{int}(A) \cap \text{int}(B) = \text{int}(A \cap B)$ ii) $\text{int}(A) \cup \text{int}(B) \subseteq \text{int}(A \cup B)$.	